# Evaluation of the torsional provisions in the 1985 NBCC

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ABSTRACT: The results of a parametric study of earthquake time history response of asymmetric single storey structures with one axis of symmetry modelled as two degree-of-freedom systems are compared with static seismic code provisions. A large number of such systems with a range of mass centre to rigidity centre eccentricities, a range of uncoupled lateral natural frequencies and a number of torsional to lateral frequency ratios $\Omega_0$ were subjected to several earthquake acceleration records. The study shows that the static approach does not give reliable estimates for the response of frames in asymmetric buildings, even when the amplification factors provided by the 1985 NBCC are incorporated into the formulation. Consequently, new formulae for torsion are proposed which are intended to improve the agreement between the time history and static results.

## 1 INTRODUCTION

It is the accepted view of the profession that building code provisions should be simple to implement yet provide conservative results. However, it is also agreed that the extent of conservatism should not vary appreciably among different structural members and configurations with similar ductility supply. Using this view as a criterion, it is of some interest to test the torsional provisions of the recently revised National Building Code of Canada (NBCC 1985).

The problems facing code writers in this area are not simple since a dynamic system having at least two degrees-of-freedom (DoF) has to be transformed for the purpose of analysis to an 'equivalent' static one, and therefore the effect of location in plan on the response of different members cannot adequately be dealt with even for a simple single storey monosymmetric system such as that shown in Figure 1. The reason for the difficulty is the dynamic modal coupling of lateral and torsional motions in asymmetric structures. Moreover, the effect of the natural vibration period has to be addressed, since the response of low period asymmetric structures differs from that of systems with moderate to high periods.

In view of these difficulties it is indeed surprising that the present code provisions are in many cases in reasonable agreement with results of dynamic time history analyses. Yet the discrepancies, some of which were already derived in an earlier study (Rutenberg and Pekau 1983), warrant some modifications to the torsional provisions of the 1985 NBCC. Some possible amendments are proposed in this paper.

This paper considers only the coupling effect in monosymmetric structures. Since the response of systems with bidirectional eccentricity has been shown to be lower (Tso 1983), the simplified model appears to be conservative. The problem of accidental eccentricity is not dealt with in this paper in view of the limited data

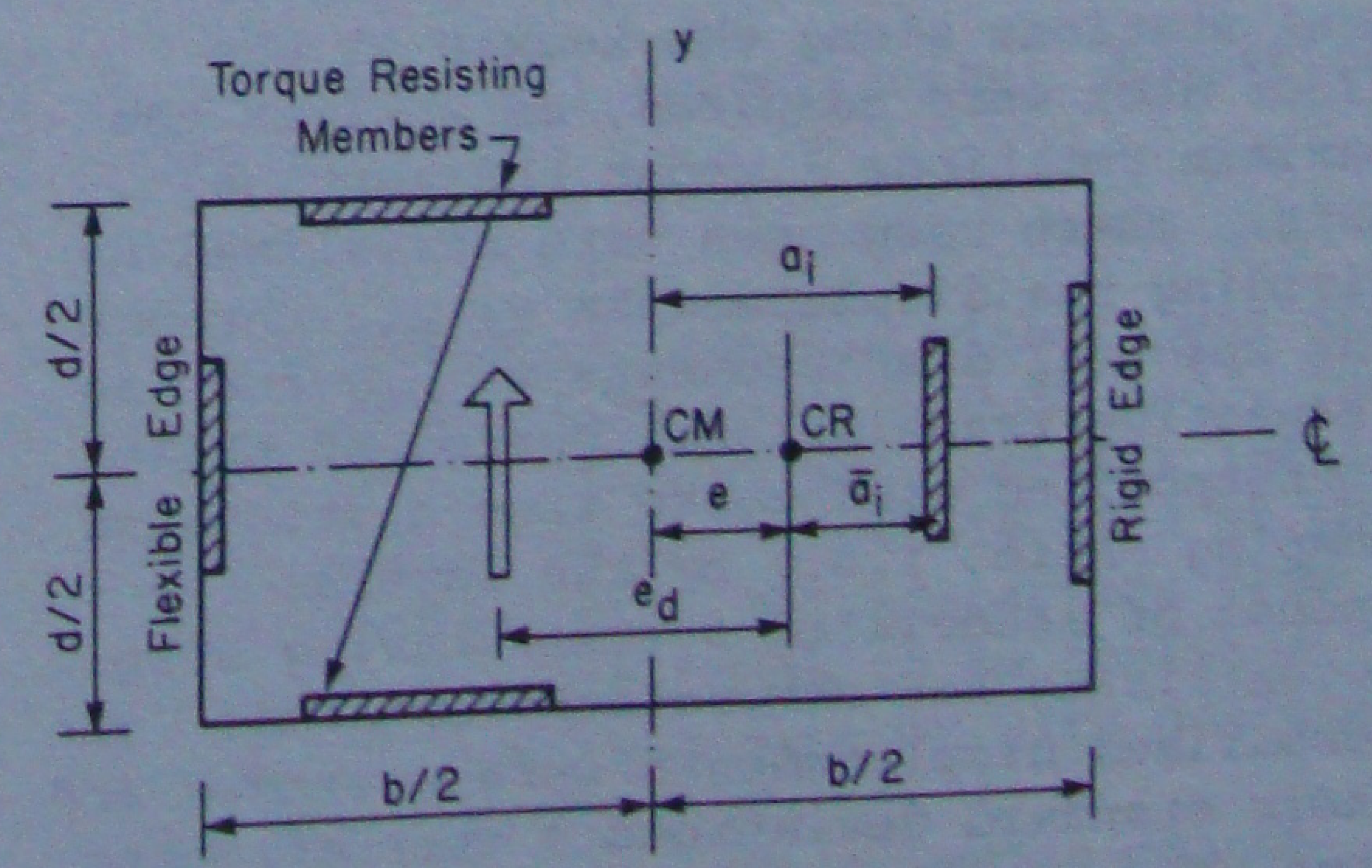


Fig. 1 Plan of single storey structural model

available on rotational ground motion, structural imperfections affecting the stiffness distribution, and possible variation in mass distribution. Some of these problems have recently been discussed by Rutenberg and Heidebrecht (1985).

## 2 PARAMETRIC STUDY

The study consisted of performing a large number of time history analyses using the computer program DRAIN 2D for a number of 2-DoF systems of the type shown in Fig. 1. These have lateral natural periods $T_o$ = 0.25, 0.50, 0.75, 1.00, 1.25, 1.50 and 2.0 secs., and eccentricity ratios $e^* = e/\rho$ = 0.1, 0.3, 0.5, 0.7, 0.9, 1.1, 1.3 and 1.5, where $\rho$ = mass radius of gyration about the mass centre CM. Note that $e = 1.5\rho$ is an unrealistically high eccentricity even for very narrow floor plans ($b \approx 3.0\rho$), and thus may be considered as an upper bound. These systems were excited by five earthquake time histories: El Centro 1934 NS, 1940 NS, 1940 EW; Olympia 1949 N80E; and Taft 1952 N69W. Five percent damping was assumed for the two coupled modes. Seven torsional to lateral frequency ratios were considered: $\Omega_0$ = 0.71, 0.87, 1.0, 1.12, 1.22, 1.41 and 1.58. Note that $\Omega_0$ is defined here as $\Omega_0 = r_{k,CR}/\rho$, i.e., the stiffness radius of gyration $r_k$ is taken about the centre of rigidity CR whereas $\rho$ is about CM. This somewhat unusual definition is based on the observation that this frequency ratio is independent of eccentricity, i.e., by assuming it to be constant while varying the eccentricity, it is possible to isolate the effect of eccentricity from all other properties of the system. With the other definitions, however, a constant frequency ratio implies that variations in eccentricity are accompanied by changes in stiffness or mass related parameters.

Maximum lateral displacements at several stations along the x-axis of the floor deck were computed, as well as the maximum rotations $\theta$ about the vertical axis. These responses were normalized by dividing the displacements $y_{i,max}$ through the spectral displacements $y_{o,max}$ for the respective translational period of the corresponding earthquake time history and damping ratio, i.e., by the lateral response of associated systems having zero eccentricity but otherwise identical. The normalized displacements were averaged and their standard deviations $\sigma$ were computed. Thus, the results represent a sample of 35 response maxima at several stations along the floor deck for realistic ranges of eccentricities and frequency ratios.

Comparisons of these results with the static method given in the 1985 NBCC were then made. Note, however, that in order to make the comparison meaningful, the code based analyses disregarded the accidental eccentricity contribution to the expression for dynamic eccentricity $e_d$. Thus, the following expressions were employed:

$$e_{d1} = 1.5e + 0.05b \tag{1}$$

for members located on the flexible side of the floor, namely the side of CM away from CR and

$$e_{d1} = 0.5e - 0.05b \tag{2}$$

for members on the rigid side. The relevant expressions in the code have 0.10b rather than 0.05b, i.e., 0.05b was taken as the accidental eccentricity contribution (Tso 1983).

The results (averages, average + 1.0$\sigma$ and extreme values) are given for three locations on the floor plan (see Fig. 1): members located at $a_i = \pm 1.5\rho$ (=0.5b) resisting shear and torque, as well as members at the two other edges oriented perpendicular to the direction of excitation and resisting torque only. Note that for the latter case the code values were computed using equation (1). Since the code itself is not explicit regarding the analysis of these members, the use of equation (1) is believed to reflect the intentions of the code writers.

Results for $a_i = -1.5\rho$, i.e., members located at the flexible edge of the deck are given in Fig. 2 for four rotational to translational frequency ratios: $\Omega_0$ = 0.71, 1.00, 1.22 and 1.58. Taking the 'average + 1.0$\sigma$' as the standard for comparison, it can be seen that for low frequency ratios representing rotationally flexible structures the NBCC formula is very conservative. It is still conservative at large eccentricities for $\Omega_0$ = 1.0, with the conservatism being reduced with increasing frequency ratio (see Figs. 2c and 2d).

Consider now the response of members at the rigid edge ($a_i = +1.5\rho$) for $\Omega_0$ = 0.71, 0.87, 1.00, 1.12, 1.22 and 1.58. Using the same standard for comparison it can be seen from Fig. 3 that, for $\Omega_0$ = 0.71, the code formula appreciably underestimates the response. This large underestimate is reduced with increasing $\Omega_0$, but the code expression becomes satisfactory only when $\Omega_0$ > 1.22 (see Figs. 3e and 3f).

The results for torque resisting members are given in Fig. 4. It is seen that, for all the frequency ratios considered, the

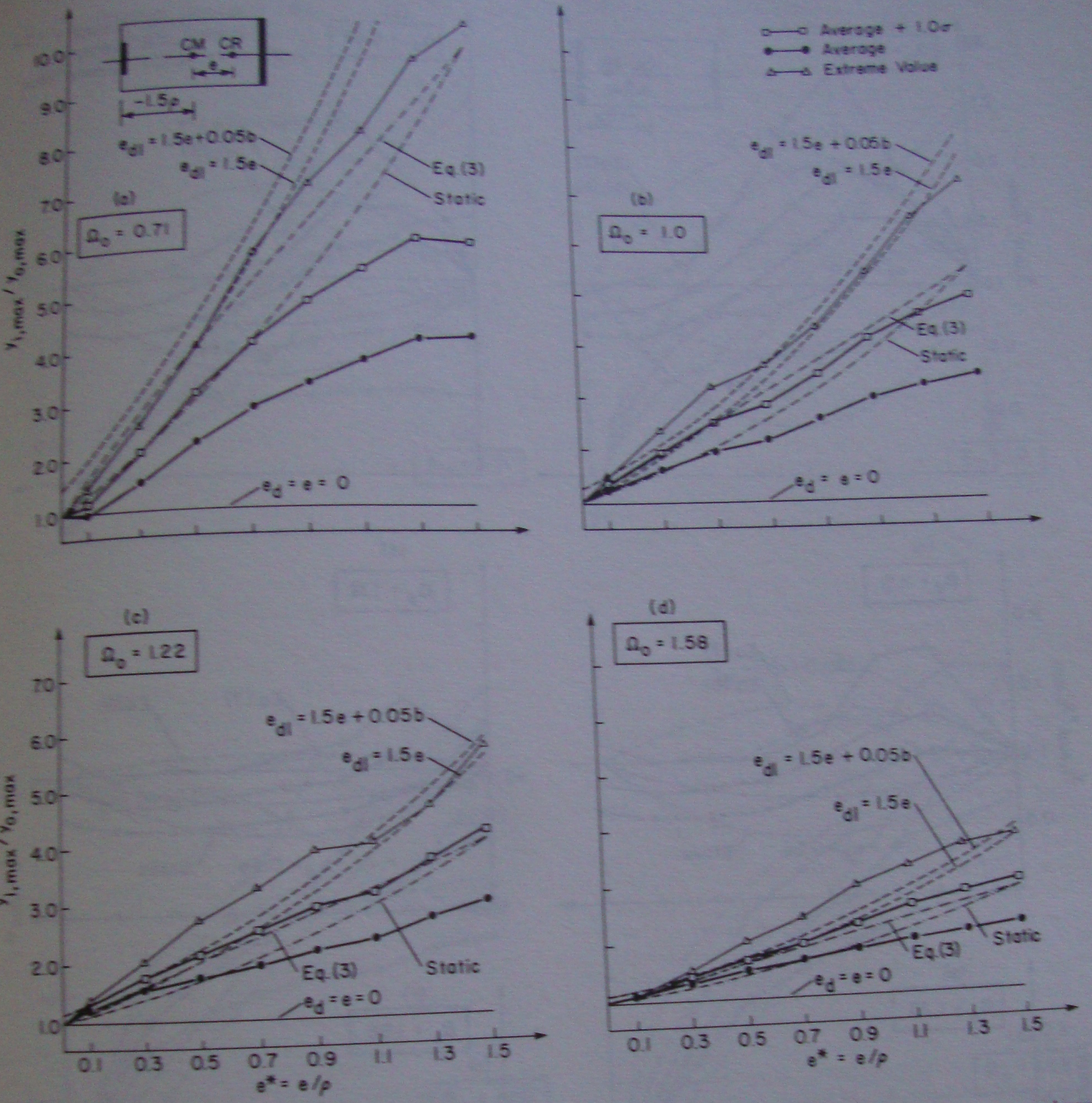


Fig. 2 Maximum displacement at flexible edge ($a_i$ = -1.5ρ) vs. eccentricity: comparison with static, code and proposed formulae: $\Omega_0$ = 0.71 to 1.58

NBCC formula, which is linear with eccentricity, underestimates the response for small eccentricities and overestimates it for large eccentricities. It is also evident that the largest discrepancies occur at very realistic eccentricities and frequency ratios (see Figs. 4b and 4c).

It will be observed that the foregoing results, which lump together all the computed data, mask the effect of the lateral vibration period $T_0$. Whereas no systematic variation of response with $T_0$ was discerned for periods to $T_0 \geq 0.50$ sec., the results for $T_0 \leq 0.25$ sec. predict larger displacements on the flexible side of the deck (see Fig. 5). This departure from the general trend tends to be confined to highly eccentric systems with decreasing frequency ratio $\Omega_0$.

In summary, the code formulae cannot satisfactorily predict the response of members located near the stiff edge of the

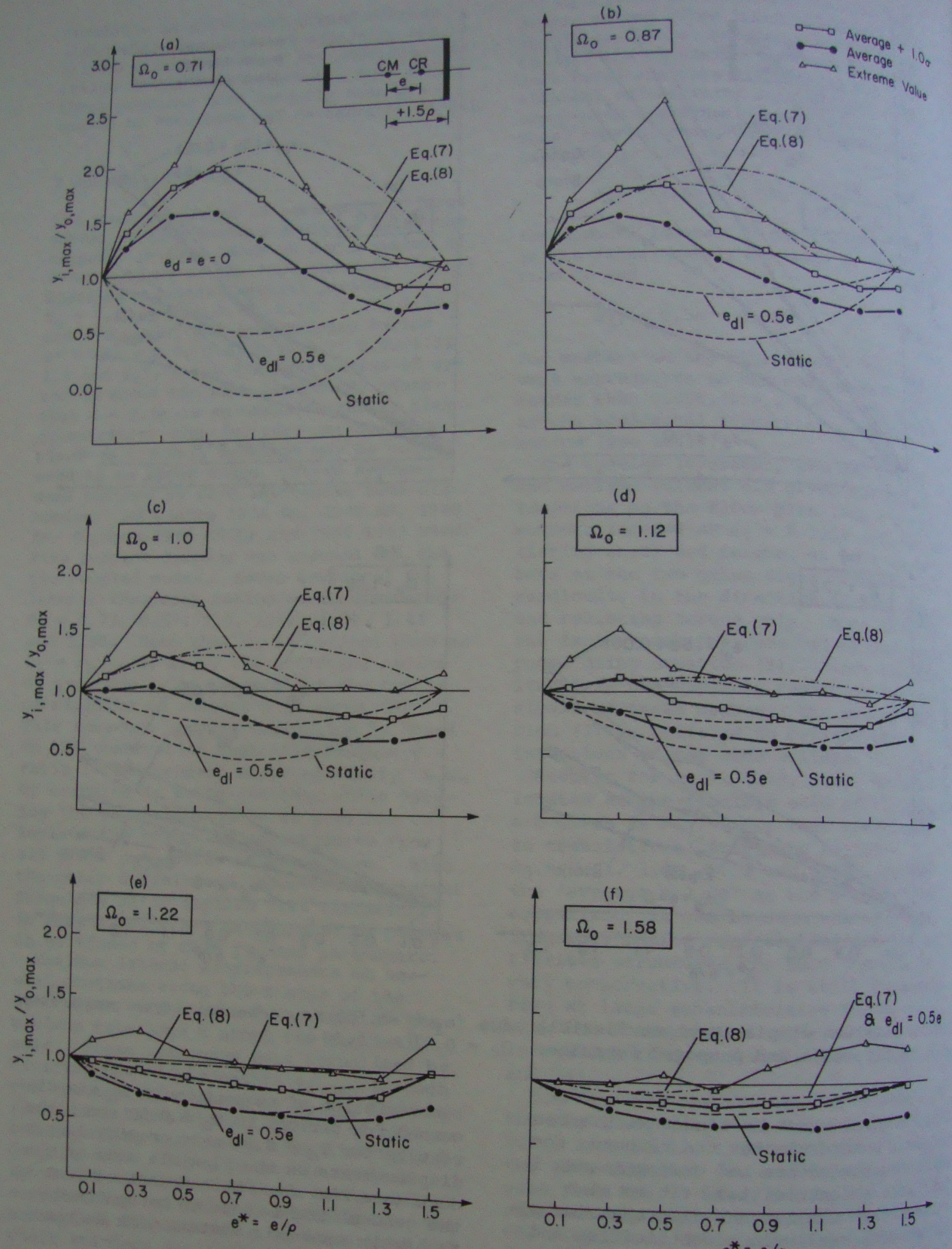


Fig. 3 Maximum displacement at rigid edge ($a_i = +1.5\rho$) vs. eccentricity: comparison with static code and proposed formulae: $\Omega_0 = 0.71$ to $1.58$

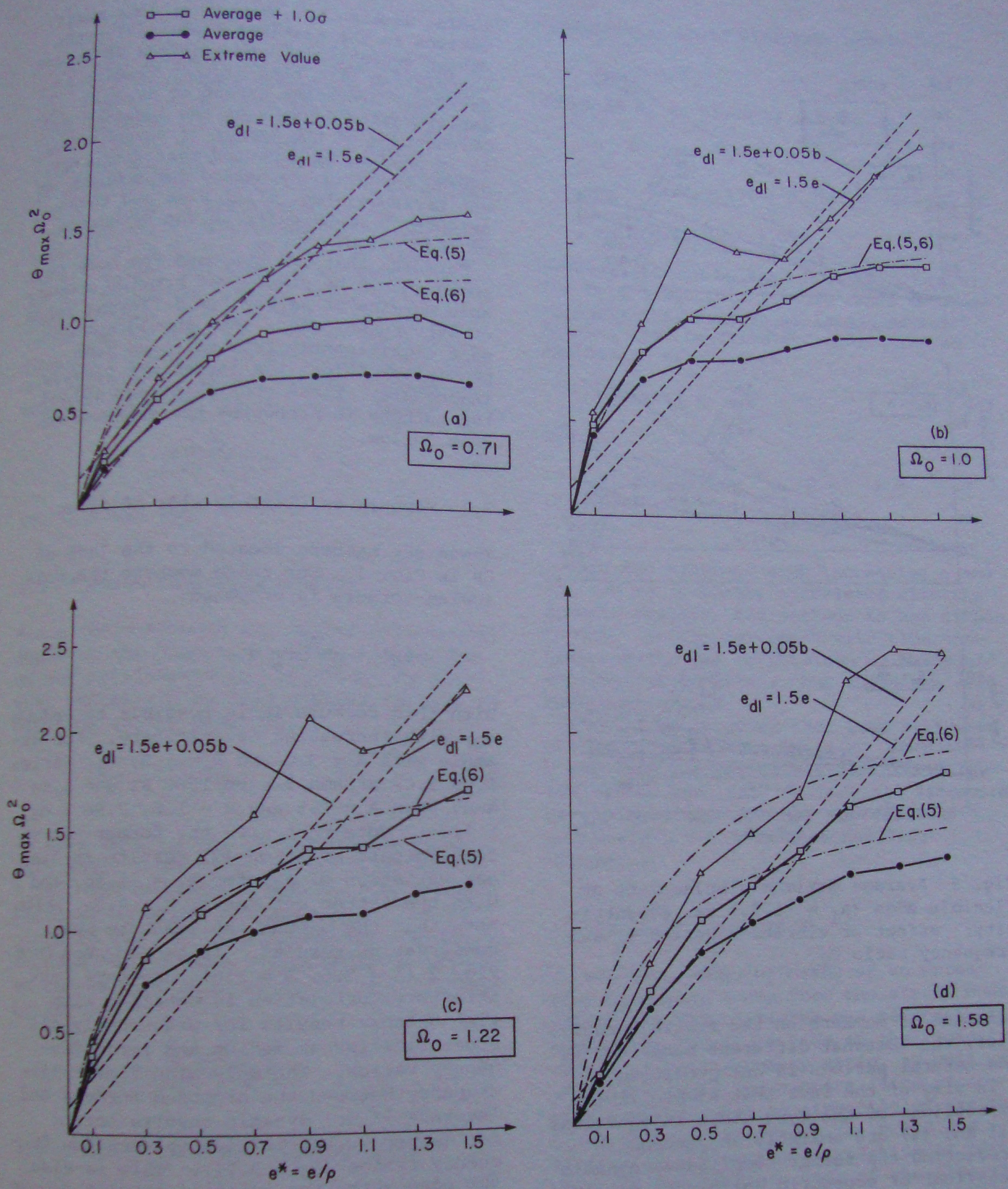


Fig. 4 Maximum rotation vs. eccentricity: comparison with code and proposed formulae: $\Omega_0$ = 0.71 to 1.58

floor nor of members resisting torque only. In these cases the results are likely to be very nonconservative. The code predictions for members on the flexible side of the floor plan are in much better agreement with time history results and they are on the whole conservative. The latter observation is not surprising since code formulae were historically calibrated on the basis of the spectral

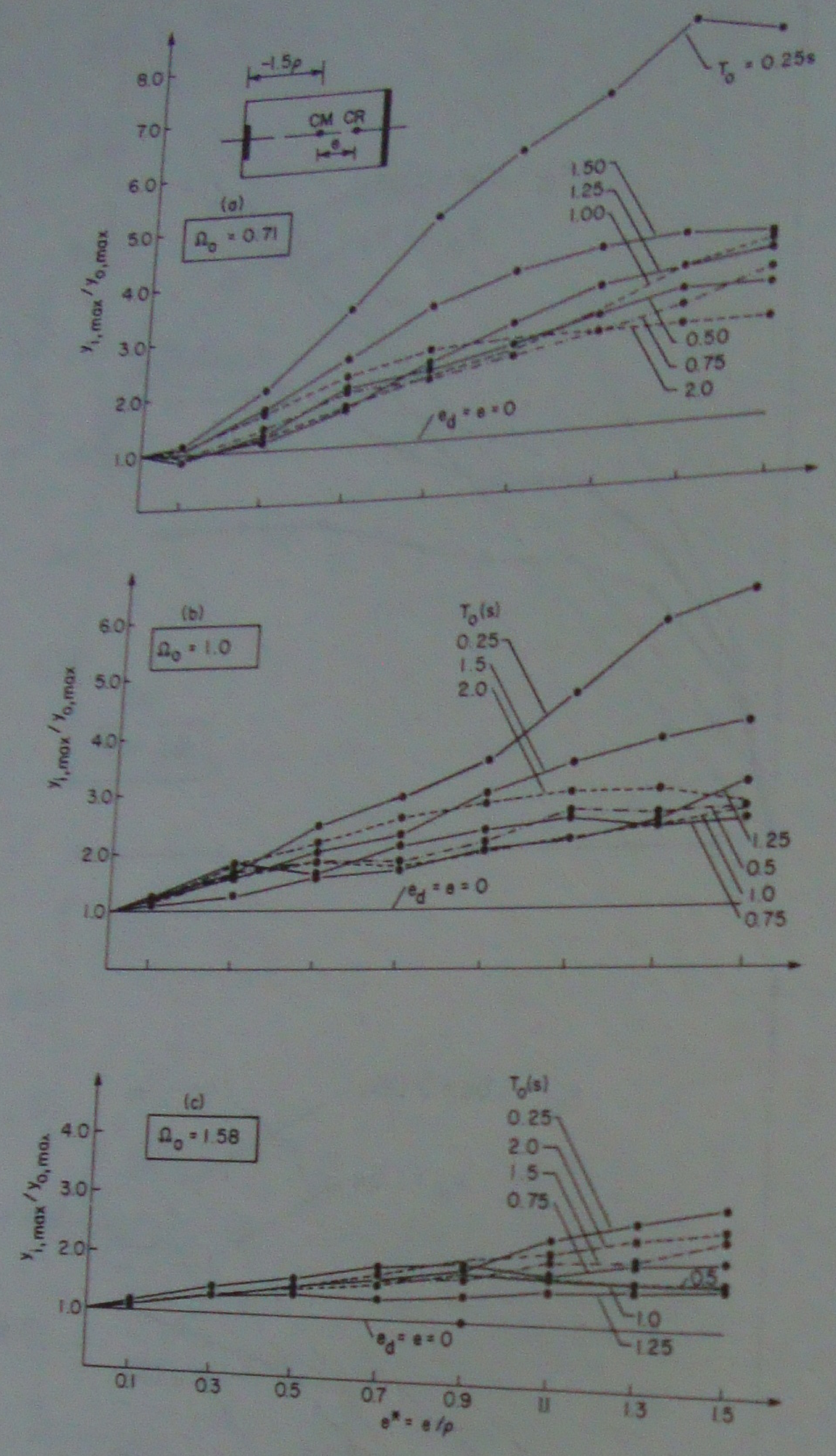


Fig. 5 Average maximum displacement at flexible edge ($a_i = -1.5\rho$) vs. eccentricity: effect of vibration period $T_o$ and frequency ratio $\Omega_0$

response of members on the flexible edge. Also, the somewhat different behaviour at low natural periods is not predicted.

In view of the fact that simple linear expressions of the type given in equations (1) and (2) are unlikely to succeed in predicting the rather complicated dynamic behaviour of eccentric buildings, possible modifications to the present code provisions are proposed in the following section and these are intended to improve the agreement between the time history and static results.

## 3 PROPOSED DESIGN FORMULAE

The parametric study described in the preceding section has shown that some modifications to the standard formulae for the design eccentricity are required if better correlation with time history results is desired. Since the extent of dynamic amplification depends on the location and orientation of the member or assemblage, it is only to be expected that different design formulae are needed for members on the flexible side, stiff side and those oriented perpendicular to the direction of excitation.

Formulae that are proposed for code purposes should be simple yet provide comparable margins of safety for different types of structures; also, they should not depart significantly from existing code provisions unless the latter are clearly inadequate. These considerations guided the authors in proposing the expressions that follow.

### 3.1 Members on flexible side of floor

These are members located to the left of CR in Fig. 1. For these members the following formula is proposed:

$$e_{d1} = \frac{A\rho}{B\rho + e}\, e \qquad (3)$$

With this formula it is possible to reduce $e_{d1}$ with increasing eccentricity. For example with A = 3.0 and B = 1.5, $e_{d1}$ varies from 2.0e at small e to 1.0e at e = 1.5ρ and, when A = 2.5 and B = 1.0, 2.5e > $e_{d1}$ > 1.0e. Note that with the former set of coefficients equation (3) results in linear variation of $e_{d1}$ for $a_i = -1.5\rho$, and with the latter set for $a_i = -1.0\rho$. Also note that the second set leads to more conservative results. It can be seen from Fig. 2 (A = 2.5, B = 1.0) that very satisfactory correlation is obtained with time history results for practically all eccentricities at medium and large frequency ratios. The only significant discrepancy between the proposed formula and 'average +1.0σ' dynamic results occurs for larger eccentricities at very low frequency ratios ($\Omega_0 = 0.71$). This is also the case where the 'static' results depart appreciably from dynamic analysis. Since designs combining very low torsional rigidity with high eccentricity are not common, and in the authors' opinion should not be encouraged, this result appears to be acceptable.

However, it is quite easy to improve the correlation at the extreme ends of the frequency range by factoring equation (3) by $\sqrt{\Omega_0}$ to read (say for A = 3.0 and B = 1.5)

$$e_{d1} = \frac{3\rho\sqrt{\Omega_0}}{1.5\rho + e}\, e \qquad (4)$$

It will be recalled that the results for $T_o = 0.25$ sec. were significantly different from those for the higher natural periods at moderate and large eccentricities, particularly at low frequency ratios. From the analysis of the dynamic results it appears that equation (3), with $A = 2.5$ and $B = 1.0$, is practically above the 'average $+1.0\sigma$' displacements of the five samples with $T_o = 0.25$ sec., at least up to $e = 0.75\rho$, and even up to higher eccentricities for $\Omega_0 \leq 0.87$. This is the range within which equation (3) is more conservative than $e_{d1} = 1.5e$. Therefore, pending a detailed study on the behaviour of laterally rigid asymmetric systems one may use equation (3) in the following form:

$$e_{d1} = \frac{2.5\rho}{1.0\rho + e}\, e \geq 1.5e \qquad (3a)$$

for $T_o < 0.5$ sec.

### 3.2 Torsion resisting members

Since these members are oriented perpendicular to the direction of excitation they do not participate in resisting the lateral forces. A formula similar in form to equation (3) is proposed:

$$e_{d1} = \frac{1.8\rho}{0.3\rho + e}\, e \qquad (5)$$

In this formula $e_{d1}$ varies from 6.0e at very small eccentricities to 1.0e at $e = 1.5\rho$. The comparison between the dynamic results, equation (5) and $e_{d1} = 1.5e$ is given in Fig. 4. Again, one may choose to improve the agreement with dynamic results at the extreme ends of the frequency range by letting

$$e_{d1} = \frac{1.8\rho\sqrt{\Omega_0}}{0.3\rho + e}\, e \qquad (6)$$

### 3.3 Members on rigid side of floor

These are members located to the right of CR in Fig. 1. The response of these members is the most difficult to describe in simple mathematical terms, as can be seen from the curves in Fig. 3.

The simplest way to improve the correlation between the code formulae and the dynamic results is to reverse the sign of the eccentricity in either the 'static' or the 0.5e results in Fig. 3 and factor them so that with increasing $\Omega_0$ the negative eccentricity would diminish, namely

$$e_{d1} = \alpha_s e \qquad (7)$$

in which $\alpha_s$ is given by

$$\alpha_s = -1.0; \quad \Omega_0 \leq 0.90$$

$$\alpha_s = -1.0+3.33(\Omega_0-0.90); \quad 0.90<\Omega_0\leq 1.35$$

$$\alpha_s = 0.5; \quad \Omega_0 > 1.35$$

However, it can be seen from Fig. 3 that, for $\Omega_0 < 1.12$, this formula tends to overestimate the response at larger eccentricities. For negative eccentricities an expression of the form

$$e_{d1} = \alpha_s^* e \qquad (8)$$

where

$$\alpha_s^* = \alpha_s(1-\Omega_0^2 e^{*3}); \quad \Omega_0^2 e^{*3}<1.0$$

$$\alpha_s = 0 \quad ; \quad \Omega_0^2 e^{*3}>1.0$$

lowers the response with increasing e and $\Omega_0$, and so is better correlated with the dynamic results, but perhaps is not simple enough. Better agreement with time history results can be obtained by means of exponential functions, but these are even more complicated.

It will be observed that some engineers prefer using natural building dimensions b and d in the design eccentricity formulae rather than the 'artificial' dimension $\rho$. In this case one may substitute $\rho \simeq D/3$ where D = the largest building dimension.

## 4 CONCLUDING REMARKS

It has been shown by means of an extensive parameter study that the static code provisions underestimate the response of members on the rigid side of the rigidity centre at low frequency ratio, as well as of torque resisting members (line of action perpendicular to direction of excitation) at inner resonance ($\Omega_0 \simeq 1.0$ and $e^* << 1.0$). Also the correlation of results on the flexible side of the floor is not always satisfactory, with overestimates at large eccentricities and at low frequency ratios. A difference between the maximum displacements and rotations of systems with uncoupled lateral period $T_o \leq 0.25$ sec. and those with higher periods for members located on the flexible side of the floor was noted. The extent of this phenomenon and its effect on the proposed design eccentricity expressions are currently under study.

It has been shown that the formulae proposed in the preceding section are in better agreement with linear time history analyses than the present static seismic code provisions. However, the usefulness of improving the code formulae may be questioned on the ground that the linear behaviour of structures designed to yield during an earthquake may not be particularly important, since ductility demand is not simply related to linear response. This view has been taken by the writers of ATC3 (1978). However, there is some evidence (Shohet 1986) to suggest that ductility demand in asymmetric structures can better be controlled when the strength of the edge members is not lower than predicted by linear analysis. Moreover, not all structures are provided with sufficient ductility, and for these better conformity with predictions based on linear dynamic analysis should be an advantage.

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